

# Generative Modeling of Turbulence

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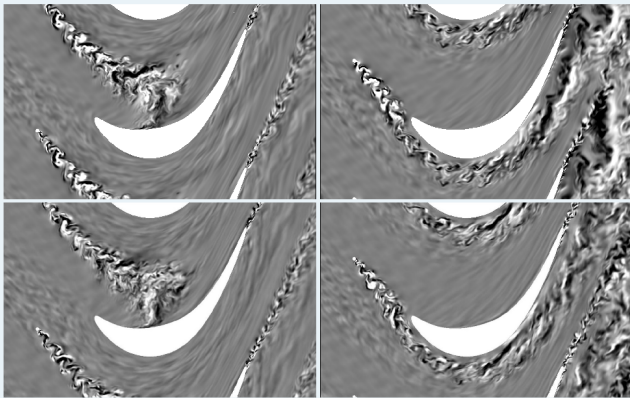
## Motivation & Contribution

Modeling turbulence is highly technical relevant but challenging in practice. Turbulent flow is inherently chaotic and develops ever smaller structures that exhibit a wide range of spatial and temporal scales. Large eddy simulations (LES) can capture large scales of turbulence but the computational costs are enormous.

We present a mathematically well founded approach for the synthetic modeling of turbulent flows using generative adversarial networks (GAN). As training data, we use fields of velocity fluctuations obtained from LES.

By investigation of physics-based metrics, we show that GAN-generated and LES flow are in excellent agreement not only at the visual level but also in their statistical properties. Thereby, GAN training and inference time significantly fall short when compared with LES. Thus, we demonstrate the ability of GAN to produce realistic turbulence while achieving a tremendous reduction of computational time.

## LPT Stator Under Periodic Wake Impact

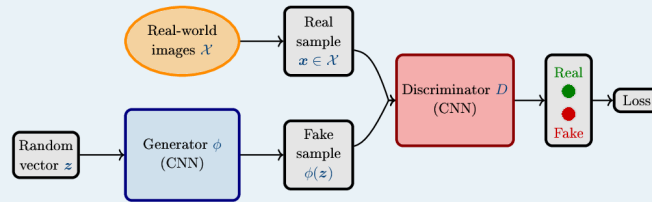


**Figure 2:** Top: Real-world images. Bottom: Synthesized images by the pix2pixHD trained on 2,000 LES images over 200 epochs with batch size 10.

## Generative Adversarial Networks (GAN)

Deep convolutional GAN (DCGAN) consist basically of two convolutional neural networks (CNN) - a generator  $\phi$  and a discriminator  $D$  which are playing a two-player minimax game described by the optimization problem

$$\min_{\phi} \max_D \mathbb{E}_{\mathbf{x} \sim \mu} [\log(D(\mathbf{x}))] + \mathbb{E}_{\mathbf{z} \sim \lambda} [\log(1 - D(\phi(\mathbf{z})))].$$



**Figure 1:** Architecture of DCGAN.

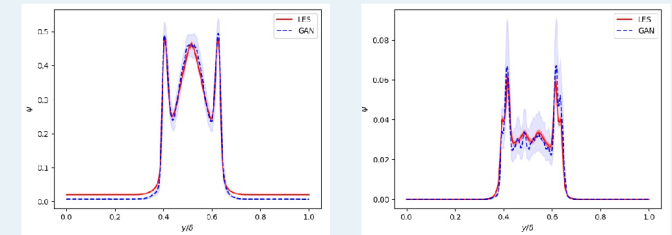
The data production process can be controlled by feeding additional information to  $D$  and  $\phi$ . The pix2pixHD is an extended version of such a conditional DCGAN framework which allows us to generate high-resolution photo-realistic images from semantic segmentation masks.

## Kármán Vortex Street (KVS)

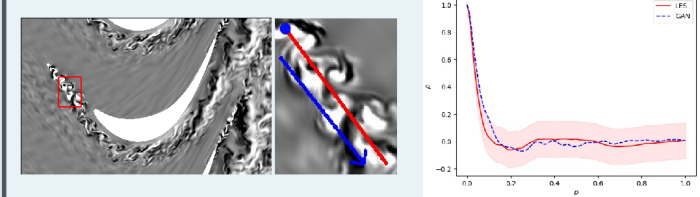


**Figure 3:** Top: Real-world images. Bottom: Synthesized images by the DCGAN trained on 5,000 LES images over 2,000 epochs with batch size 20.

## Physics-based evaluation



**Figure 4:** Mean pixel values (left) and statistical fluctuation of the deviation from mean velocities (right) for 5,000 images of the KVS along the  $y$ -axis.



**Figure 5:** Pointwise correlation for 225 images of the LPT stator along the  $x$ -axis (right) over a selected area (left).

## Computational Costs

	LES	GAN-Training	GAN-Inference
Machine	560 CPU cores of Intel Xeon "Sky-lake" Gold 6132 @2.6 GHz	GPU Quadro RTX 8000 with 48 GB	
KVS (DCGAN)	72 core weeks $\approx$ 1 day (for 5,000 images)	1.5 min/epoch ( $\approx$ 2 days for 2,000 epochs)	0.001 sec/image ( $\approx$ 5 sec for 5,000 images)
LPT stator (pix2pixHD)	640 core weeks $\approx$ 8 days (for 2,250 images)	17 min/epoch ( $\approx$ 2 days for 200 epochs)	0.01 sec/image ( $\approx$ 22.5 sec for 2,250 images)

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