Identifying Top-k Players in Cooperative Games via Shapley Bandits

Patrick Kolpaczki, Paderborn University   Viktor Bengs, Paderborn University
Eyke Hüllermeier, University of Munich (LMU)

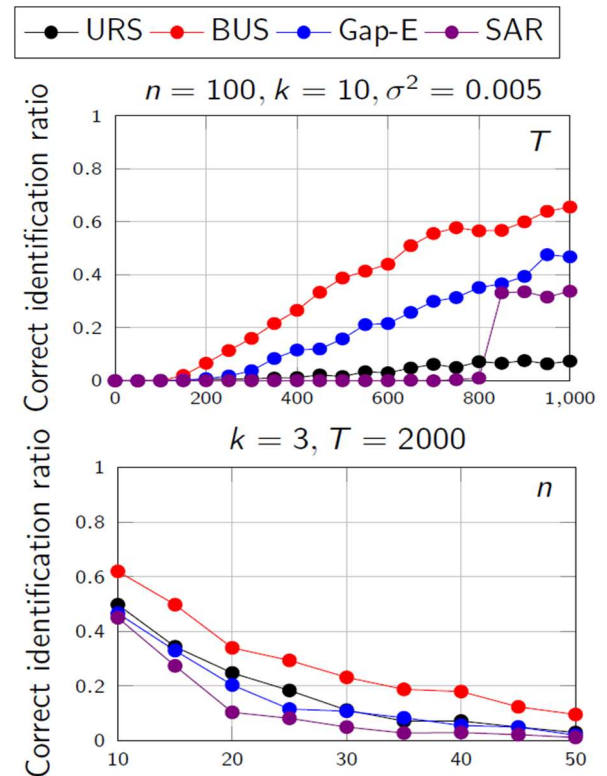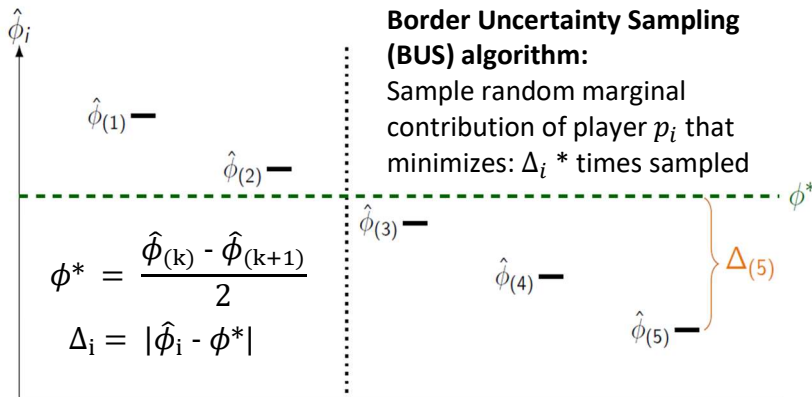**Cooperative Game:** Players can form coalitions to accomplish a task and gain a collective benefit

Coalition S	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
Worth $\nu(S)$	0	0	0	0	40	80	120	150

Key Question: How distribute the total worth among individual players?

Shapley value for player i:
$$\phi_i = \sum_{S \subseteq N \setminus \{p_i\}} \frac{1}{n \cdot \binom{n-1}{|S|}} \cdot \underbrace{\left[\nu(S \cup \{p_i\}) - \nu(S) \right]}_{\text{marginal contribution}}$$

Top-k Shapley Problem

- Given: cooperative game (N, ν)

- Access to ν is *costly*: **only one** access to ν per time step.

- Shapley values $\phi_1, \ldots, \phi_n$ are unknown.

- Players can be ordered such that $\phi_{(1)} \geq \ldots \geq \phi_{(n)}$.

- **Task:** identify the top-k players $p_{(1)}, \ldots, p_{(k)}$ for given $k \in \mathbb{N}$.

- **Fixed budget setting:** given T many accesses to ν, maximize the probability of correct identification.

Border Uncertainty Sampling (BUS) algorithm:
Sample random marginal contribution of player p_i that minimizes: Δ_i * times sampled

$\hat{\phi}_i$

$\hat{\phi}_{(1)}$ —

$\hat{\phi}_{(2)}$ —

ϕ^*

$\hat{\phi}_{(3)}$ —

$$\phi^* = \frac{\hat{\phi}_{(k)} - \hat{\phi}_{(k+1)}}{2}$$

$\hat{\phi}_{(4)}$ — $\Delta_{(5)}$

$$\Delta_i = |\hat{\phi}_i - \phi^*|$$

$\hat{\phi}_{(5)}$ —

Correct identification ratio — URS — BUS — Gap-E — SAR

$n = 100, k = 10, \sigma^2 = 0.005$ — T

$k = 3, T = 2000$ — n